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CHAIN RATIO TYPE ESTIMATOR FOR POPULATION MEAN USING KNOWN COEFFICIENT OF VARIATION OF THE STUDY CHARACTER IN THE PRESENCE OF NON- RESPONSE

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ABSTRACT

Using the known coefficient of variation of the study character, chain ratio type estimator for population mean in the presence of non-response have been proposed and their properties have been studied. The conditions under which the proposed estimator is more efficient than the relevant estimators have been obtained. The empirical studies have been given in the support of the problems in the case of positive and negative correlation between the study and the auxiliary characters which show the increase in the efficiency of the proposed estimator using known coefficient of variation of the study character with respect to the relevant estimators.

KEYWORDS: Ratio Estimator, Population Mean, Study Variable, Coefficient of Variation, Non- Response

INTRODUCTION

The use of auxiliary information in sample surveys during the stage of planning, designing, selection of units and devising the estimation procedure has been considered mainly in the field of Agricultural, Biological, Medical and Social Sciences. The reviews of the research works related to these areas have been carried out by Tripathi et al. (1994) and Khare (2003).

Das and Tripathi (1980) have proposed improved estimators for population mean using known coefficient of variation of auxiliary character. The use of coefficient of variation of the study character in proposing the estimator for population mean of the study character has been made by Searls (1964,1967) and Sen (1978). In the case of non response in the selected sample from the population, Hansen and Hurwitz (1946) have proposed the method of sub sampling from non respondents in the sample. Using Hansen and Hurwitz (1946) techniques, Khare and Srivastava (1993,95) have proposed two phase sampling estimators for population mean in the presence of non response. Khare and Kumar (2009) have proposed the estimators utilizing the coefficient of variation of the study character in the estimation of population mean using auxiliary character in the presence of non response.

In the present paper, we have proposed chain ratio type estimator for the population mean using known coefficient of variation of the study character in the presence of non response. The properties of their proposed estimator have been studied and comparative studies of the estimators have been made with the relevant estimators. In case of fixed sample sizes (n and n') and fixed cost $C \le C_0$. The empirical studies have been given in the support of the proposed estimators used in the case of positive correlation as well as negative correlation between study and auxiliary characters.

THE ESTIMATORS

Let (y,x) be the study character and auxiliary character observed in the population $U:(U_1,U_2,\ldots,U_N)$ of the size N having population means $(\bar{Y},\bar{X})$ and coefficient of variations(C_y,C_x). $(\bar{y},\bar{x})$ denote the sample means of (y,x) based on a sample of size n drawn from the population using SRSWOR method of sampling. In the case of non-response in the selected sample of size n for the study character y, we observe that n_1 units respond and n_2 units do not respond. Further, a sub sample of size $r=\frac{n_2}{K}(K>1)$ is drawn from n_2 non-responding units by making extra efforts. Hansen and Hurwitz (1946) proposed an estimator for $\bar{Y}$ which is given as follows:

$$\bar{y}^*=\frac{n_1}{n}\bar{y}_1+\frac{n_2}{n}\bar{y}_2' \tag{1}$$

$$MSE(\bar{y}^*)=\frac{f}{n}S_y^2+\frac{W_2(K-1)}{n}S_{y(2)}^2, \tag{2}$$

where $\bar{y}_1$ and $\bar{y}_2$ denote the sample means based on n_1 responding and n_2 non-responding units in the sample of size n such that $\bar{y}=\frac{n_1}{n}\bar{y}_1+\frac{n_2}{n}\bar{y}_2'$ and $\bar{y}_2'$ denotes the sample mean based on r units selected from n_2 non-responding units on y. S_y^2 and $S_{y(2)}^2$ denote the population mean squares for the whole population and for the non-responding part of the population for the study character y.

In case, when $\bar{X}$ is unknown, we select a large sample of size $n'(>n)$ from the population of size N by using SRSWOR method of sampling and we estimate $\bar{X}$ by $\bar{x}'=\frac{1}{n'}\sum_{i=1}^{n'}x_i$ and again draw a sub-sample of size n and observed y character.

In this case conventional and alternative two phase sampling ratio type estimators suggested by Khare and Srivastava (2010). In two phase sampling regression estimator for population mean $\bar{Y}$ using one auxiliary character in the presence of non-response have been proposed by Khare and Srivastava (1995) and generalized chain ratio type estimator for the population mean in the presence of non response have been proposed by Khare et al.(2011).which are given as follows:

$$T_1=\bar{y}^*\left(\frac{\bar{x}^*}{\bar{x}'}\right)^{\alpha} \tag{3}$$

$$T_2=\bar{y}^*+b^*\left(\bar{x}'-\bar{x}^*\right) \tag{4}$$

$$T_3=\bar{y}^*\left(\frac{\bar{x}^*}{\bar{x}'}\right)^{\alpha_1}\left(\frac{\bar{z}'}{\bar{Z}}\right)^{\alpha_2} \tag{5}$$

where

α,α_1 and α_2 are constants.

$$\bar{x}^*=\frac{n_1}{n}\bar{x}_1+\frac{n_2}{n}\bar{x}_2',\ \bar{x}=\frac{1}{n}\sum_{j=1}^{n}x_j,\ \bar{x}'=\frac{1}{n'}\sum_{j=1}^{n'}x_j,\ b^*=\frac{\hat{S}_{yx}}{s_x^2}\ \text{and}\ s_x^2=\frac{1}{n-1}\sum_{i=1}^{n}(x_i-\bar{x})^2.$$

Searls (1964) proposed an estimator $\bar{y}^{**} = a\bar{y}^{*}$ for $\bar{Y}$ in the presence of non response, where a is a constant. The value of a, for which the $MSE(\bar{y}^{**})$ will be minimum is given by:

$$a_{(opt.)} = [1 + \frac{f}{n}\frac{S_y^2}{\bar{Y}^2} + \frac{W_2(K-1)}{n}\frac{S_{y(2)}^2}{\bar{Y}^2}]^{-1} \quad (6)$$

Since $\frac{S_y^2}{\bar{Y}^2}$ and $\frac{S_{y(2)}^2}{\bar{Y}^2}$ do not differ significantly, so, we may approximate $\frac{S_{y(2)}^2}{\bar{Y}^2} = C_y^2$ and neglecting the terms of order $\frac{1}{N}$, we have

$$\hat{a}_{(opt)} = \left[1 + \frac{C_y^2}{n}(1 + \frac{n_2}{n}(K-1))\right]^{-1} \quad (7)$$

Now, we define an estimator $\bar{y}^{**}$ for $\bar{Y}$ by improving the estimator $\bar{y}^{*}$ using Searls(1964) method, which is given as follows:

$$\bar{y}^{**} = [1 + \frac{C_y^2}{n}(1 + \frac{n_2}{n}(K-1))]^{-1}\bar{y}^{*} \quad (8)$$

The mean square error (MSE) of $\bar{y}^{**}$ is given as follows:

$$MSE(\bar{y}^{**}) = (1-A)\frac{S_y^2}{n} + (1-2B)\frac{W_2(K-1)}{n}S_{y(2)}^2 \quad (9)$$

where $A = \frac{C_y^2}{n}[1 - W_2^2(K-1)^2]$ and $B = \frac{C_y^2}{n}[1 + W_2(K-1)]$.

Comparing $MSE(\bar{y}^{**})$ with MSE $(\bar{y}^{*})$, we see that

$$MSE(\bar{y}^{**}) < MSE(\bar{y}^{*}) \text{ when } 1 < a < 1 + \frac{1}{W_2} \quad (10)$$

Under this condition, the estimator may be improved by replacing a better estimator $\bar{y}^{**}$ of $\bar{Y}$ than $\bar{y}^{*}$ in case of non-response in sample survey. However the estimator $\bar{y}^{**}$ may also be more efficient than $\bar{y}^{*}$ beyond the range of $a > 1 + \frac{1}{W_2}$.

Now, using known coefficient of variation of the study character, we propose chain ratio type estimator for population mean in the presence of non-response, which is given as follows:

$$T_R = \bar{y}^{**}\left(\frac{\bar{x}^{*}}{\bar{x}'}\right)^p\left(\frac{\bar{z}'}{\bar{Z}}\right)^q \quad (11)$$

where $\bar{y}^{**} = a\bar{y}^{*}$, p and q are constants.

In order to derive the expressions for the mean square error of the estimators:

Let $\bar{y}^{*} = \bar{Y}(1+\epsilon_0^{*}), \bar{x}^{*} = \bar{X}(1+\epsilon_1^{*}), \bar{x}' = \bar{X}(1+\epsilon_1'), \bar{x} = \bar{X}(1+\varepsilon_1)$ and $\bar{z}' = \bar{Z}(1+\epsilon_2')$ such that $E(\epsilon_0^{*}) = E(\epsilon_1^{*}) = E(\epsilon_1') = E(\epsilon_2') = E(\epsilon_1) = 0, |\epsilon_0^{*}|, |\epsilon_1^{*}|, |\epsilon_1'|, |\epsilon_2'|, |\epsilon_1| < 1.$

By using simple random sampling without replacement method of sampling, we get,

$$E(\epsilon_0^{*2}) = \frac{1}{\bar{Y}^2}V(\bar{y}^*) = \frac{1}{\bar{Y}^2}\left\{\left(\frac{1}{n}-\frac{1}{N}\right)S_y^2 + \frac{W_2(K-1)}{n}S_{y(2)}^2\right\}$$

$$E(\epsilon_1^{*2}) = \frac{1}{\bar{X}^2}V(\bar{x}^*) = \frac{1}{\bar{X}^2}\left\{\left(\frac{1}{n}-\frac{1}{N}\right)S_x^2 + \frac{W_2(K-1)}{n}S_{x(2)}^2\right\}$$

$$E\left(\epsilon_1^{'2}\right) = \frac{1}{\bar{X}^2}V(\bar{x}^{'}) = \frac{1}{\bar{X}^2}\left(\frac{1}{n^{'}}-\frac{1}{N}\right)S_x^2$$

$$E\left(\epsilon_2^{'2}\right) = \frac{1}{\bar{Z}^2}V(\bar{z}^{'}) = \frac{1}{\bar{Z}^2}\left(\frac{1}{n^{'}}-\frac{1}{N}\right)S_z^2$$

$$E(\epsilon_1^2) = \frac{1}{\bar{X}^2}V(\bar{x}) = \frac{1}{\bar{X}^2}\left(\frac{1}{n^{'}}-\frac{1}{N}\right)S_x^2$$

$$E(\epsilon_0^*,\epsilon_1^*) = \frac{1}{\bar{Y}\bar{X}}COV(\bar{y}^*,\bar{x}^*) = \frac{1}{\bar{Y}\bar{X}}\left\{\left(\frac{1}{n}-\frac{1}{N}\right)S_{yx} + \frac{W_2(K-1)}{n}S_{yx(2)}\right\}$$

$$E(\epsilon_0^*,\epsilon_1^{'}) = \frac{1}{\bar{Y}\bar{X}}COV(\bar{y}^*,\bar{x}^{'}) = \frac{1}{\bar{Y}\bar{X}}\left(\frac{1}{n^{'}}-\frac{1}{N}\right)S_{yx}$$

$$E(\epsilon_0^*,\epsilon_2^{'}) = \frac{1}{\bar{Y}\bar{Z}}COV(\bar{y}^*,\bar{z}^{'}) = \frac{1}{\bar{Y}\bar{Z}}\left(\frac{1}{n^{'}}-\frac{1}{N}\right)S_{yz}$$

$$E(\epsilon_1^*,\epsilon_2^{'}) = \frac{1}{\bar{X}\bar{Z}}COV(\bar{x}^*,\bar{z}^{'}) = \frac{1}{\bar{X}\bar{Z}}\left(\frac{1}{n^{'}}-\frac{1}{N}\right)S_{xz}$$

$$E(\epsilon_1^*,\epsilon_1^{'}) = \frac{1}{\bar{X}^2}COV(\bar{x}^*,\bar{x}^{'}) = \frac{1}{\bar{X}^2}V(\bar{x}^{'}) = \frac{1}{\bar{X}^2}\left(\frac{1}{n^{'}}-\frac{1}{N}\right)S_x^2$$

$$E(\epsilon_1^{'},\epsilon_2^{'}) = \frac{1}{\bar{X}\bar{Z}}COV(\bar{x}^{'},\bar{z}^{'}) = \frac{1}{\bar{X}\bar{Z}}\left(\frac{1}{n^{'}}-\frac{1}{N}\right)S_{xz}$$

The contribution of the terms involving the powers in $\epsilon_0^*, \epsilon_1^*, \epsilon_1^{'}, \varepsilon_1$ and $\epsilon_2^{'}$ of order higher than two in mean square errors are assumed to be negligible. So, the expressions for the MSE of the proposed estimator and relevant estimators up to the terms of order n^{-1} are given as follows:

MEAN SQUARE ERROR OF THE PROPOSED ESTIMATOR

$$MSE(T_R) = a^2V(\bar{y}^*) + \bar{Y}^2[(a-1)^2 + A_1\{a^2p(2p-1)C_x^2 + 2ap(2a-1)C_{yx}\} - A_2\{aq(2aq-q-a+1)C_z^2 + 2aq(2a-1)C_{yz}\} + A_3\{a^2p(2p-1)C_{x(2)}^2 + 2ap(2a-1)C_{yx(2)}\}] \quad (12)$$

where $a_{(opt.)} = [1 + \frac{f}{n}\frac{S_y^2}{\bar{Y}^2} + \frac{W_2(K-1)}{n}\frac{S_{y(2)}^2}{\bar{Y}^2}]^{-1}$,

$$p_{(opt.)} = \frac{A_1\{2(1-2a)C_{yx} + (a-1)C_x^2\} + A_3\{2(1-2a)C_{yx(2)} + (a-1)C_{x(2)}^2\}}{2A_1(2a-1)C_x^2 + 2A_3(2a-1)C_{x(2)}^2},$$

$$q_{(opt.)} = \frac{2(1-2a)C_{yz} + (a-1)C_z^2}{2(2a-1)C_z^2}$$

Mean square errors of the estimators T_1, T_2 and T_3 are given as follows:

$$MSE(T_1) = V(\bar{y}^*) + \bar{Y}^2\left[A_1\left(\alpha^2 C_x^2 + 2\alpha C_{yx}\right) + A_3\left(\alpha^2 C_{x(2)}^2 + 2\alpha C_{yx(2)}\right)\right] \tag{13}$$

$$MSE(T_2) = V\left(\bar{y}^*\right) - \bar{Y}^2\left[\left(\frac{1}{n} - \frac{1}{n'}\right)\rho_{yx}^2 C_y^2 - \frac{W_2(k-1)}{n}\left\{B_1^2 C_{x(2)}^2 - 2B_1 C_{yx(2)}\right\}\right] \tag{14}$$

$$MSE(T_3) = V\left(\bar{y}^*\right) + \bar{Y}^2\left[A_1\left\{\alpha_1(2\alpha_1 - 1)C_x^2 + 2\alpha_1 C_{yx}\right\} - A_2\left\{\alpha_2^2 C_z^2 + +2\alpha_2 C_{yz}\right\} + A_3\left\{\alpha_1(2\alpha_1 - 1)C_{x(2)}^2 + 2\alpha_1 C_{yx(2)}\right\}\right] \tag{15}$$

Where, $V(\bar{y}^*) = \bar{Y}^2\left\{\frac{f}{n}C_y^2 + \frac{W_2(k-1)}{n}C_{y(2)}^2\right\}$, $\alpha_{(opt)} = \alpha_{1(opt)} = -\frac{A_1 C_{yx} + A_3 C_{yx(2)}}{A_1 C_x^2 + A_3 C_{x(2)}^2}$, $\alpha_{2(opt.)} = -\frac{C_{yz}}{C_z^2}$,

$B_1 = \frac{\bar{Y}\rho_{yx}C_y}{\bar{X}C_x}$ $A_1 = \left(\frac{1}{n} - \frac{1}{n'}\right)$, $A_2 = \left(\frac{1}{n'} - \frac{1}{N}\right)$ and $A_3 = \frac{W_2(k-1)}{n}$

$\rho_{yx(2)} = \frac{s_{yx(2)}}{s_{y(2)}s_{x(2)}}$ and $(S_{yx(2)}, \rho_{yx(2)})$ denote the covariance and correlation between y and x characters for the non response group of the population.

DETERMINATION OF n', n and k FOR THE FIXED COST $C \le C_0$

Let us assume that C_0 be the total cost (fixed) of the survey apart from overhead cost. The expected total cost of the survey apart from overhead cost is given as follows:

$$C = (g_1' + g_2')n' + n\left(g_1 + g_2 W_1 + g_3\frac{W_2}{k}\right), \tag{16}$$

where

g_1' : the cost per unit of obtaining information on auxiliary character x at the first phase.

g_2' : the cost per unit of obtaining information on additional auxiliary character z at the first phase.

g_1 : the cost per unit of mailing questionnaire/visiting the unit at the second phase.

g_2 : the cost per unit of collecting, processing data obtained from n_1 responding units.

g_3 : the cost per unit of obtaining and processing data (after extra efforts) for the sub sampling units.

The expression for, $MSE(T_R)$ can be expressed in terms of G_0, G_1, G_2 and G_3 which are the coefficients $\frac{1}{n}$, $\frac{1}{n'}$

, $\frac{k}{n}$ and $\frac{1}{N}$ respectively. The expression of $MSE(T_R)$ is given as follows:

$$MSE(T_R)_{\min} = \frac{G_0}{n} + \frac{G_1}{n'} + \frac{k\,G_2}{n} - \frac{G_3}{N} \tag{17}$$

For obtaining the optimum values of n', n, k for the fixed cost $C \le C_0$, we define a function ϕ which is given as:

$$\phi = MSE(T_R)_{\min} + \lambda(C - C_0), \tag{18}$$

where λ is the Lagrange's multiplier.

We differentiating ϕ with respect to n', n, k and equating zero, we get optimum values of n', n and k which are given as follows:

$$n'_{opt} = \sqrt{\frac{G_1}{\lambda(g'_1 + g'_2)}}\ , \tag{19}$$

$$n_{opt} = \sqrt{\frac{(G_0 + k_{opt}G_2)}{\lambda\left(g_1 + g_2W_1 + g_3\dfrac{W_2}{k_{opt}}\right)}} \tag{20}$$

and

$$k_{opt} = \sqrt{\frac{G_0 g_3 W_2}{G_2(g_1 + g_2W_1)}}\ , \tag{21}$$

where

$$\sqrt{\lambda} = \frac{1}{C_0}\left[\sqrt{G_1(g'_1 + g'_2)} + \sqrt{(G_0 + k_{opt}G_2)\left(g_1 + g_2W_1 + g_3\frac{W_2}{k_{opt}}\right)}\right] \tag{22}$$

The minimum value of $MSE(T_R)$ for the optimum values of n', n and k in the expression $MSE(T_R)$, we get:

$$MSE(T_R) = \frac{1}{C_0}\left[\sqrt{G_1(g'_1 + g'_2)} + \sqrt{(G_0 + k_{opt}G_2)\left(g_1 + g_2W_1 + g_3\frac{W_2}{k_{opt}}\right)}\right]^2 - \frac{G_3}{N}\ , \tag{23}$$

Now neglecting the term of O (N^{-1}), we have

$$MSE(T_R)_{\min} = \frac{1}{C_0}\left[\sqrt{G_1(g'_1 + g'_2)} + \sqrt{(G_0 + k_{opt}G_2)\left(g_1 + g_2W_1 + g_3\frac{W_2}{k_{opt}}\right)}\right]^2 \tag{24}$$

AN EMPIRICAL STUDY

To illustrate the results we considered the data earlier consider by Khare and Sinha (2009).The description of the population is given below:

96 village wise population of rural area under Police-station – Singur, District -Hooghly, West Bengal has been taken under the study from the District Census Handbook 1981. The 25% villages (i.e. 24 villages) whose area is greater than 160 hectares have been considered as non-response group of the population. The number of agricultural laboures in the village is taken as study character (y) while the area (in hectares) of the village, the number of cultivators in the village and the total population of the village are taken as auxiliary characters x and z respectively.

The values of the parameters of the population under study are as follows:

$\bar{Y}=137.9271$, $\bar{X}=144.8720$, $\bar{X}=185.2188$, $S_y=182.5012$, $C_x=.8115$, $S_{y(2)}=287.4202$, $C_{x(2)}=.9408$, $C_z=1.0529$, $C_{z(2)}=1.4876$, $\rho_{yx}=0.773$, $\rho_{yx(2)}=0.724$, $\rho_{yz}=0.786$, $\rho_{yz(2)}=0.787$, $\rho_{xz}=0.819$, $\rho_{xz(2)}=0.724$, $W_2=0.25$, $N=96$, $n=24$, $n'=60$.

Table 1: Relative Efficiency (In %) of the Estimators With Respect to $\bar{y}^*$ for the Fixed Values of n', n and Different Values of k (N =96, n' =60 and n =24)

Estimators	$1/k$		
	1/4	1/3	1/2
$\bar{y}^*$	100 (34.59)*	100 (29.99)	100 (25.40)
T_1	140(24.65)	150(20.05)	164 (15.45)
T_2	131 (26.47)	141 (21.26)	158 (16.06)
T_3	178 (19.39)	176 (17.08)	173(14.70)
T_R	197 (17.58)	194 (15.46)	191 (13.27)

*Figures in parenthesis give the MSE (.).

From table 1, we obtained that for fixed sample sizes (n', n), the proposed estimators T_R is more efficient in comparison to the efficiency of the estimators $\bar{y}^*$, T_1, T_2 and T_3.

Table 2: Relative Efficiency (In %) of the Estimators with Respect to $\bar{y}^*$ (for the Fixed Cost $C \le C_0$ =Rs.220, c_1' =Rs. 0.90, c_2'=Rs. 0.10, c_1=Rs. 2, c_2=Rs. 4, c_3=Rs. 25).

Estimators	k_{opt}	n'_{opt}	n_{opt}	Efficiency
$\bar{y}^*$	2.49	---	27	100 (0.5372)*
T_1	2.70	56	21	120 (0.4480)
T_2	1.77	76	16	151 (0.3555)
T_3	1.52	72	14	158 (0.3240)
T_R	1.01	74	13	182 (0.2948)

*Figures in parenthesis give the MSE (.).

From table 2, we obtained that for the fixed cost $C \leq C_0$ the proposed estimators T_R is more efficient in comparison to the estimators $\bar{y}^*$, T_1, T_2 and T_3.

CONCLUSIONS

By making use of coefficient of variation the proposed estimator is more efficient then relevant estimators in case of fixed sample sizes (n and n') and also in case of fixed cost $C \leq C_0$.

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